Essays on Operation Management: Information Disclosure and Inventory Control

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Road Map

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- Designing business operations in services
- Controlling the process of production

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This thesis addresses two specific topics:

- Designing business operations in services: Quality Disclosure
- Controlling the process of production: Inventory Control
  - Multi-echelon
  - Single-echelon with a minimum order quantity and batch ordering

Essay I

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Quality Disclosure for Experience Goods with Boundedly Rational Customers

Motivation

- Quality of experience goods impacts customer willingness to pay and firm profits.
- Firms know their quality information better than potential customers.
  - Firms (e.g., retailers or service providers) have a variety of market research instruments.
    - Customer feedbacks, market surveys, market data
    - Expert evaluations
    - Medical data in health care
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    - Potential customers lack of resources and expertise to access reliable quality information.
- Information asymmetry $\rightarrow$ market inefficiency
- A natural question to ask is ...

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However, is this the real case in practice?

Boundedly Rational Customers

1. Customers may not hold rational expectations due to a variety of reasons.
    - lack of the technical/ necessary expertise (due to limited education or time)
    - unintentional ignorance, e.g., complex disclosure format
    - mandatory or costless disclosure, e.g., annual financial reporting
    - unverified information
2. Customers tend to rely on anecdotes from those who have bought the product or have experienced the service previously via several channels.
    - online retailers: Amazon, EBay, Expedia, Taobao
    - independent third-party websites: yelp.com, tripadvisor.com
    - social media: Facebook, Twitter, Weibo, Wechat

“*Indeed, other consumers’ experiences constitute a key input for those who have not yet experienced the product or service*” (Hu et al. 2015).

Research Questions

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Problem

What is the impact of the customer bounded rationality on a firm's quality information disclosure decision? When should firms voluntarily disclose quality information, and when should not?

Model Setup

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To model customer bounded rationality, I adopt the anecdotal reasoning framework (Osborne and Rubinstein 1998; Spiegler 2006).

- $S(N)$ Framework: each individual consumer obtains $N (N = 1, 2, 3, ...)$ anecdotes or samples about a service realization in the past.
- Assume that each consumer "combines" multiple samples by simply taking the sample average.
- Example: 5 samples (3 high, 2 low), $\tilde{\alpha} = 3/5$.

Optimal Decision: Disclosure vs. Nondisclosure

Proposition

There exists $\underline{\alpha}, \overline{\alpha}$ *such that* $0 < \underline{\alpha} \leq \overline{\alpha} < 1$ *and it is optimal to adopt the nondisclosure strategy for* $\alpha \in [0, \underline{\alpha}] \cup [\overline{\alpha}, 1]$, *while it may be optimal to adopt the voluntary disclosure strategy for some* $\alpha \in [\underline{\alpha}, \overline{\alpha}]$.

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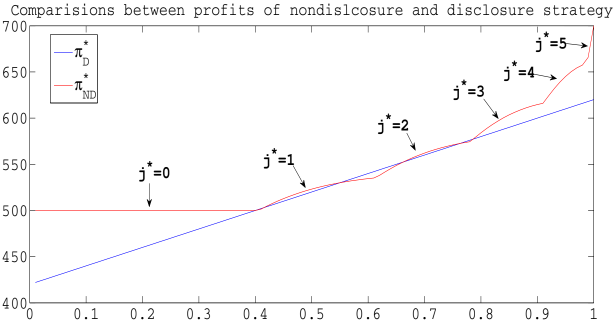
- Conventional wisdom with customer full rationality: threshold policy for positive disclosure cost. (Jovanovic, 1982)
- The optimal disclosure strategy can be quite complicated.

Optimal Decision: Disclosure vs. Non-disclosure



Optimal Decisions(N=5, $v_H = 7$, $v_L = 5$, $\overline{\lambda}$=100 and K=80)

α	[0,0.40]	[0.41,0.41]	[0.42,0.55]	[0.56,0.61]	[0.62,0.66]
j^*	0	1	1	1	2
OPT	ND	D	ND	D	D

α	[0.67,0.76]	[0.77.0.78]	[0.79,0.91]	[0.92,0.98]	[0.99,1]
j^*	2	2	3	4	5
OPT	ND	D	ND	ND	ND

OPT: Optimal; ND: Non-Disclosure ; D: Disclosure

Key Insights

- Two forces driving the profitability of nondisclosure.
  - Advantage of nondisclosure: pricing flexibility – choose from multiple candidate prices as individual customers may receive different samples.
  - Disadvantage of nondisclosure: variability of customers' quality perceptions – some customers may underestimate its quality, whereas others may overestimate it.

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- Two forces driving the profitability of nondisclosure.
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  - Disadvantage of nondisclosure: variability of customers' quality perceptions – some customers may underestimate its quality, whereas others may overestimate it.
- When the quality level is high or low, the anecdotes and samples can deliver a similar level of information to customers as a counterpart under quality information disclosure. (due to binomial distribution)
- When the quality level is medium, the information received by customers may be noisy. Therefore, to reduce the impact of market inefficiency, the firm may have an incentive to disclose its quality information.

Empirical Findings

- Surprisingly, the results are consistent with some recently empirical findings. Luca and Smith (2015) find that **mid-ranked business schools are most likely to disclose their rankings**.

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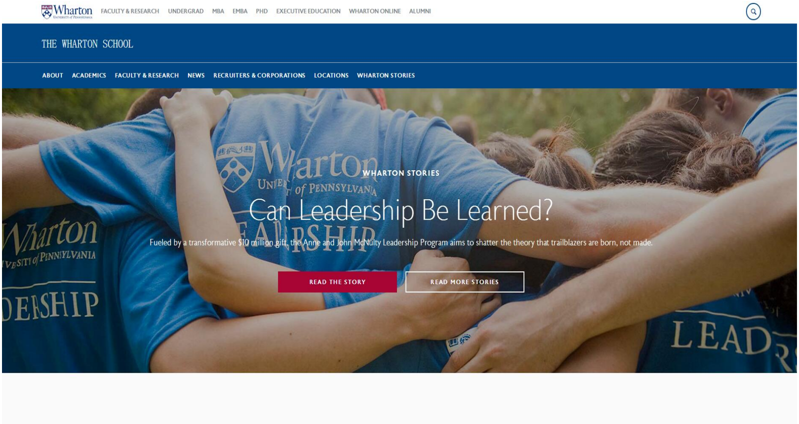

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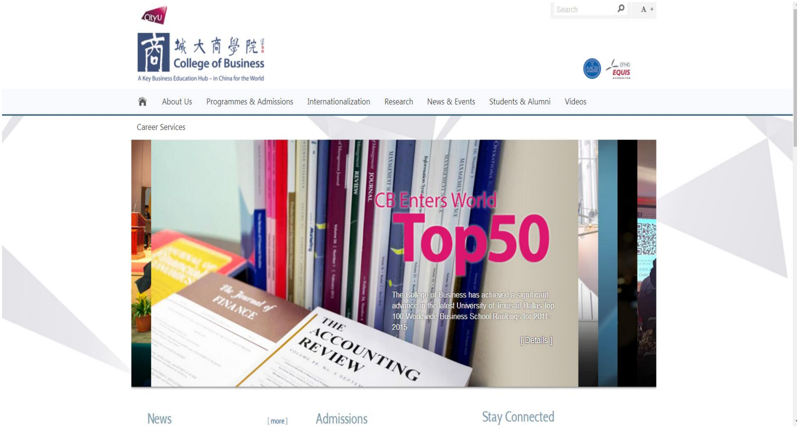

Conclusion and Discussion

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- The findings underscore the importance of customer bounded rationality–the firm should be more careful in deciding its quality disclosure strategy.
- The results also provide a new explanation on why the unraveling result fails to hold in practice.

Essay II

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Performance Bounds and Asymptotic Optimality of Modified (r, Q) Policies for Stochastic Distribution Inventory Systems

Distribution System

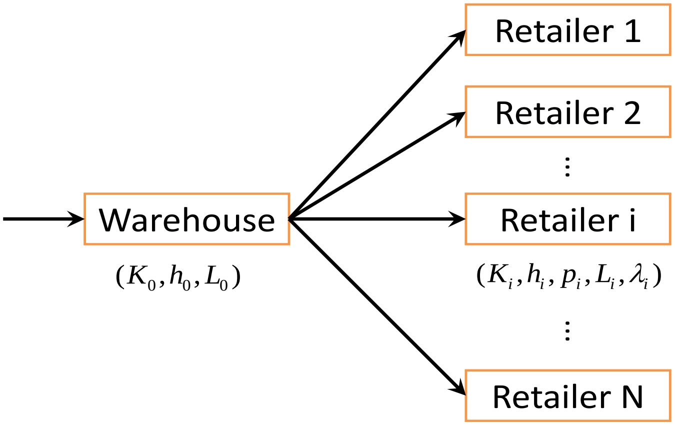

Figure: Distribution System

Literature Review-Bounds and Heuristics

- Hu and Yang (2014): a serial system
  - Modified echelon (r, Q) policy;
  - $1 + K_1/K_2$-optimal;
  - An alternative performance bound dependent on optimal solutions of single-stage systems.

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- Distribution: Axsater (1993); Chen and Zheng (1997); Shang et al. (2015)...
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  - Provide easy-to-implement heuristics:(r,nQ) policy and (S,T) policy.
  - Numerically show that the heuristics perform well.

★ No matter how extensively the numerical experiments have been conducted, there are no guarantees with respect to the heuristics performance on other problem instances.

★ It is necessary to identify a guaranteed worst-case optimality gap.

Definition of Modified (r, Q) Policy

In practice, the (r, Q) policy is widely used for multi-echelon inventory systems, because of

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Motivated by Hu and Yang (2014) and the wide application of (r, Q) policy, I propose the following policy for distribution systems.

Definition (Modified Echelon $(\mathbf{r}, \mathbf{Q})$ Policy for Distribution Systems)

The upstream installation ships to downstream installation on the basis of its observation of the echelon inventory position at the downstream installation. In particular, if the echelon inventory position at Installation i is at or below r_i and the upstream has positive on-hand inventory, then a shipment is sent to Installation i to raise its echelon inventory position as close as possible to $r_i + Q_i$.

Performance Evaluation of Modified (r, Q) Policy

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2. Specifically, I intend to identify the relative gap between the long-run average cost under a given modified echelon $(\mathbf{r},\mathbf{Q})$ policy, $C(\mathbf{r},\mathbf{Q})$, and the optimal cost $C^*_{\mathscr{B}}$, which can be expressed as

$$RG(\mathbf{r},\mathbf{Q}) \equiv (C(\mathbf{r},\mathbf{Q}) - C^*_{\mathscr{B}})/C^*_{\mathscr{B}}.$$

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$$RG(\mathbf{r}, \mathbf{Q}) \leq (C(\mathbf{r}, \mathbf{Q}) - C^{*Lower}_{\mathscr{B}}) / C^{*Lower}_{\mathscr{B}}.$$
4. It is also difficult to calculate $C(\mathbf{r}, \mathbf{Q})$. Therefore, I need an upper bound on $C(\mathbf{r}, \mathbf{Q})$ itself.

An Upper Bound of $C(\mathbf{r},\mathbf{Q})$

Theorem

For any given modified echelon $(\mathbf{r},\mathbf{Q})$ policy, the long-run average system-wide cost has an upper bound: $C(\mathbf{r},\mathbf{Q}) \leq \sum_{i=1}^{N} C_i(r_i,Q_i) + \hat{C}_0(r_0,Q_0) + \sum_{i=1}^{N} \lambda_0 K_i / Q_0$, where

$$\hat{G}_i(y) \equiv \bar{\Gamma}_i(y) - C_i(r_i,Q_i)$$

$$\Lambda_0(y) \equiv \mathbb{E}[h_0(y-D_0) + \sum_{i=1}^{N} \hat{G}_i(y-D_0)]$$

$$\hat{C}_0(r_0,Q_0) \equiv \frac{1}{Q_0}\left[\lambda_0 K_0 + \int_{r_0}^{r_0+Q_0} \Lambda_0(y)dy\right]$$

A Specific $(\hat{\mathbf{r}}, \hat{\mathbf{Q}})$ Policy

By comparing the lower and upper bound, I choose...

1. $r_i = r_i^*$ and $Q_i = Q_i^*$ that minimize $C_i(r_i, Q_i)$ ($C_i^* \equiv C_i(r_i^*, Q_i^*)$).

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2. Specifically, I solve the following single-stage optimization problem:

$$\min_{r_0, Q_0} \tilde{C}_0(r_0, Q_0) \equiv \min_{r_0, Q_0} \frac{1}{Q_0}\left[\lambda_0(K_0 + \sum_{i=1}^{N} K_i) + \int_{r_0}^{r_0+Q_0} \Lambda_0(y)dy\right]$$

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3. Define $(\tilde{r}_0^*, \tilde{Q}_0^*) \equiv \arg\min_{r_0, Q_0} \tilde{C}_0(r_0, Q_0)$. $\tilde{C}_0^* \equiv \tilde{C}_0(\tilde{r}_0^*, \tilde{Q}_0^*)$

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4. We construct a heuristic modified echelon $(\mathbf{r}, \mathbf{Q})$ policy as follows:

$$(\hat{\mathbf{r}}, \hat{\mathbf{Q}}) = (\hat{r}_0, \hat{Q}_0, \hat{r}_1, \hat{Q}_1, \ldots, \hat{r}_N, \hat{Q}_N) = (\tilde{r}_0^*. \tilde{Q}_0^*, r_1^*, Q_1^*, \ldots, r_N^*, Q_N^*) \quad (MERQD)$$

Performance Bound

Theorem

The modified echelon $(\hat{\mathbf{r}}, \hat{\mathbf{Q}})$ policy in (*MERQD*) is at least $1+(\tilde{C}_0^* - C_0^*)/(\sum_{i=1}^N C_i^* + C_0^*)$-optimal, i.e., $\frac{\sum_{i=1}^N C_i^* + \tilde{C}_0^*}{\sum_{i=1}^N C_i^* + C_0^*}$-optimal.

- C_i^*, C_0^* and $\tilde{C}_0^*$ are the optimal costs for single-stage inventory systems, and thus can be easily computed as shown in Zheng (1992).

Theorem

The modified echelon $(\hat{\mathbf{r}}, \hat{\mathbf{Q}})$ policy in (*MERQD*) is at least $\max\{\sqrt{\frac{\lambda_0}{2\beta_1\beta_2\lambda_m} + \frac{1}{4}} + \frac{1}{2}, \frac{1}{\beta_2}\}$-optimal, where $m \in \arg\max_{i=1,\ldots,N}\{K_i\}$, $\beta_1 \equiv \hat{Q}_0^*/Q_m^*$ and $\beta_2 \equiv C_0^*/\hat{C}_0^* \leq 1$.

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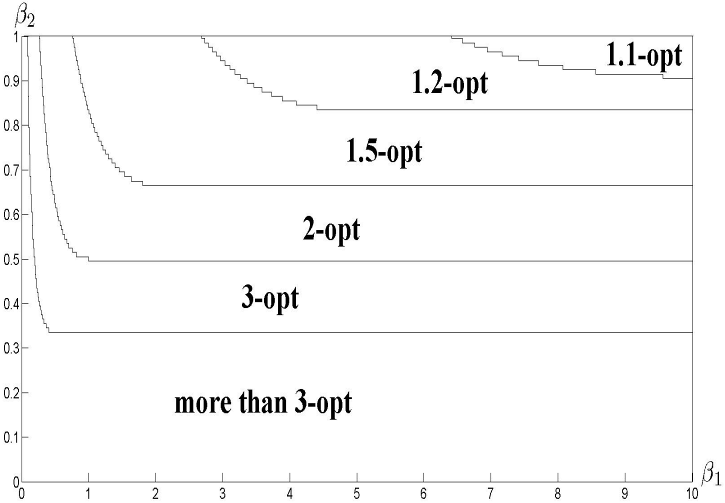

$$\max\{1+\sqrt{\frac{\lambda_0}{2\beta_1\beta_2\lambda_m}+\frac{1}{4}}-\frac{1}{2},\frac{1}{\beta_2}\} \text{ with } \lambda_0/\lambda_m=2.$$

Asymptotic Optimality

Theorem

The modified echelon $(\hat{\mathbf{r}}, \hat{\mathbf{Q}})$ policy in (MERQD) is asymptotically optimal if for any $m \in \arg\max_{i=1,\dots,N}\{K_i\}$, one of the following conditions holds:
(i) $K_m > 0$ and $K_0/K_m \to \infty$.
(ii) $h_0/h_m \to 0$.
(iii) $h_0/p_m \to 0$.

Numerical Examples

The complete test of primitive values is given by: $L_0 \in \{0,1,2\}$, $L_1 \in \{0,1,2\}$, $K_0 \in \{100,200,600\}$, $K_1 \in \{10,20,40\}$, $h_0 \in \{0.05,0.1,0.2\}$, $h_1 \in \{0.3,0.5,1\}$, $p_1 \in \{3,5,10\}$, $\lambda_1 \in \{3,5,7\}$ with other primitives fixed as $N = 2$, $L_2 = 1$, $K_2 = 20$, $h_2 = 0.5$, $p_2 = 5$, $\lambda_2 = 5$.

	δ_1	δ_4
Average (%)	10.84	6.21
Standard deviation (%)	5.31	3.20
Minimum (%)	3.37	1.39
Maximum (%)	33.83	21.65

Table: Overall performance of modified echelon $(\hat{\mathbf{r}}, \hat{\mathbf{Q}})$ policy

$\delta_1 = (UB - C_{\mathscr{B}}^{*Lower})/C_{\mathscr{B}}^{*Lower}$

$\delta_4 = (RC - C_{\mathscr{B}}^{*Lower})/C_{\mathscr{B}}^{*Lower}$, RC: Real Cost obtained by simulation

Contribution

The contribution:

- on the technical side: I fill the gap in the literature on distribution systems by developing a heuristic policy with a performance guarantee.
- on the implication side: the bounds and asymptotic results demonstrate the robustness of single-stage (r, Q) inventory policies.

Essay III

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Effective Inventory Control Policies with a Minimum Order Quantity and Batch Ordering

Introduction

- Minimum Order Quantity (MOQ)
  - Order Quantity $\geq$ MOQ
- Batch Ordering
  - Order Quantity= $n\times$ batch size

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Research Questions

- Integration of MOQ and batch ordering
    - What kind of policy should be used to manage the inventory in such a system ?
    - What is the optimal policy parameter(s)?

(s, k) policy

- the (s, k) policy: given an initial inventory position x_t and two integer parameters s and k, where $s < k < s + M$, the order quantity q_t is

$$q_t = y_t - x_t = \begin{cases} M + mQ, & \text{if } x_t \leq s; \\ M, & \text{if } s < x_t \leq k; \\ 0, & \text{if } x_t > k. \end{cases}$$

where $m \geq 1$, and m is the unique integer such that $s + M < y_t \leq s + M + Q$.

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- $\{y_i\}$ is a discrete time Markov chain. The unique steady state probabilities $\vec{\pi} = \{\pi_1, \pi_2, ..., \pi_M\}$ exist.

$$\begin{cases} \sum_{i=1}^{M} \pi_i = 1 \\ \vec{\pi} P = \vec{\pi} \end{cases}$$

- The long-run average cost: $\sum_{i=1}^{M} \pi_i C(k+i)$.

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$$q_t = y_t - x_t = \begin{cases} M + mQ, & \text{if } x_t \leq s; \\ M, & \text{if } s < x_t \leq k; \\ 0, & \text{if } x_t > k. \end{cases}$$

  where $m \geq 1$, and m is the unique integer such that $s + M < y_t \leq s + M + Q$.

- $\{y_i\}$ is a discrete time Markov chain. The unique steady state probabilities $\vec{\pi} = \{\pi_1, \pi_2, ..., \pi_M\}$ exist.

$$\begin{cases} \sum_{i=1}^{M} \pi_i = 1 \\ \vec{\pi} P = \vec{\pi} \end{cases}$$

- The long-run average cost: $\sum_{i=1}^{M} \pi_i C(k+i)$.

Proposition

k^* satisfies $y^* - M \leq k^* < y^* \leq k^* + M$.

S policy

- The order quantity in period t is

$$q_t = y_t - x_t = \begin{cases} M + mQ, & \text{if } x_t \le S - M; \\ M, & \text{if } S - M < x_t < S; \\ 0, & \text{if } x_t \ge S. \end{cases}$$

- A special case of the (s,k) policy with $s = S - M$ and $k = S - 1$ ($\Delta = M - 1$).
- The simple (R,Q) policy, when there is no MOQ constraint, i.e., $M = 0$.
- The policy of Kiesmuller et al. (2011), when $Q = 1$.

Numerical Examples

- Demand Distribution
  - Truncated normal distribution
  - Poisson distribution
- Two Other Policies
  - the optimal cost ———value iteration (Bertsekas 2005, vol 2)
    the minimal long-run average cost among all admissible policies (as period increases, the long-run average cost converges to a constant.)
  - the cost of the optimal (s,S) policy with $s = S - M$

h=1;	$M = 30$;	$Q \in \{3,5,6,10,15\}$;
$\mathbb{E}(D) \in \{10,15,20,30,40\}$; $p/(h+p) \in \{0,80,0.85,0.90,0.95\}$;		
$c.v. \in \{0.1,0.2,0.3,0.4\}$ for normal distribution		

Table: Base parameter values for the numerical experiments

$$G_1 = \frac{C_{s,k} - C_{OPT}}{C_{OPT}} * 100\% \qquad G_2 = \frac{C_{s,S} - C_{s,k}}{C_{s,k}} * 100\%$$

Factor	Value	avg G_1	max G_1	avg G_2	min G_2	max G_2
Q	3	1.43	25.11	37.15	1.11	150.98
	5	1.32	24.32	41.76	3.75	182.92
	6	1.29	23.52	44.68	7.23	235.18
	10	0.91	20.33	54.72	25.65	266.56
	15	0.71	19.03	59.45	30.18	228.50
$\mathbb{E}(D)$	10	0.10	2.12	29.56	16.50	49.49
	15	0.52	5.63	34.56	17.83	53.79
	20	0.71	6.05	27.82	1.11	70.01
	30	4.29	25.11	66.52	29.27	150.98
	40	0.04	0.20	79.28	18.40	266.56
c.v.	0.1	4.41	25.11	72.31	1.11	266.56
	0.2	0.08	1.12	53.40	18.26	112.41
	0.3	0.01	0.01	36.72	17.95	58.03
	0.4	0.03	0.20	27.77	16.50	42.91
p/(h+p)	0.8	0.70	13.59	49.23	4.20	228.50
	0.85	0.96	16.87	48.51	2.19	266.56
	0.9	1.28	20.86	46.94	1.11	255.01
	0.95	1.59	25.11	45.52	1.56	235.18

Conclusion and Future Directions

- The heuristic policies perform well in comparison with other policies:
  - have a close performance to the optimal policy in most cases;
  - outperform the (s, S) policy.

Conclusion and Future Directions

- The heuristic policies perform well in comparison with other policies:
  - have a close performance to the optimal policy in most cases;
  - outperform the (s, S) policy.
- Future directions:
  - heuristics with performance bounds;
  - multi-echelon systems with MOQ (and batch ordering).

Thank You!